

# Transverse Spin Physics

## Lecture I

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# The plan:

- **Lecture I:**

Transverse spin structure of the nucleon  
Overview of past experiments  
History of interpretation  
Overview of present understanding

- **Lecture II**

Transverse Momentum Dependent distributions (TMDs)  
Sivers function  
Twist-3

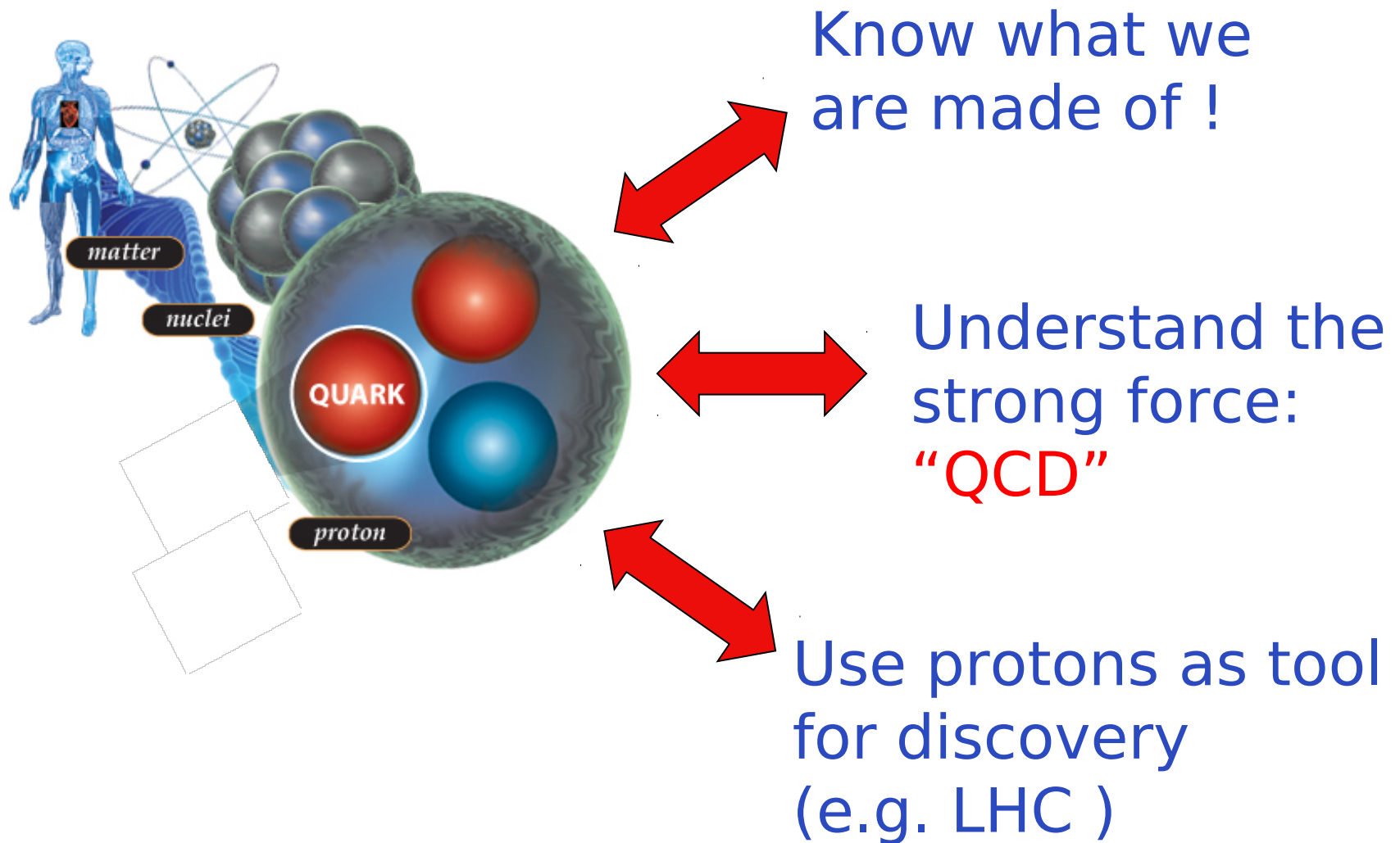
- **Lecture III**

Transversity  
Collins Fragmentation Function  
Global analysis

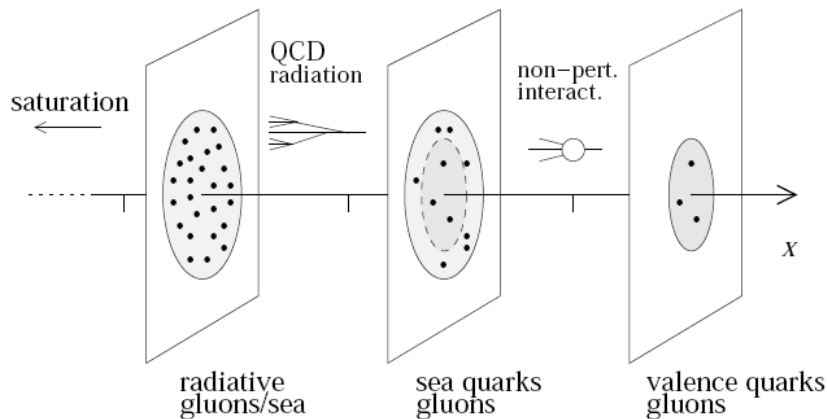
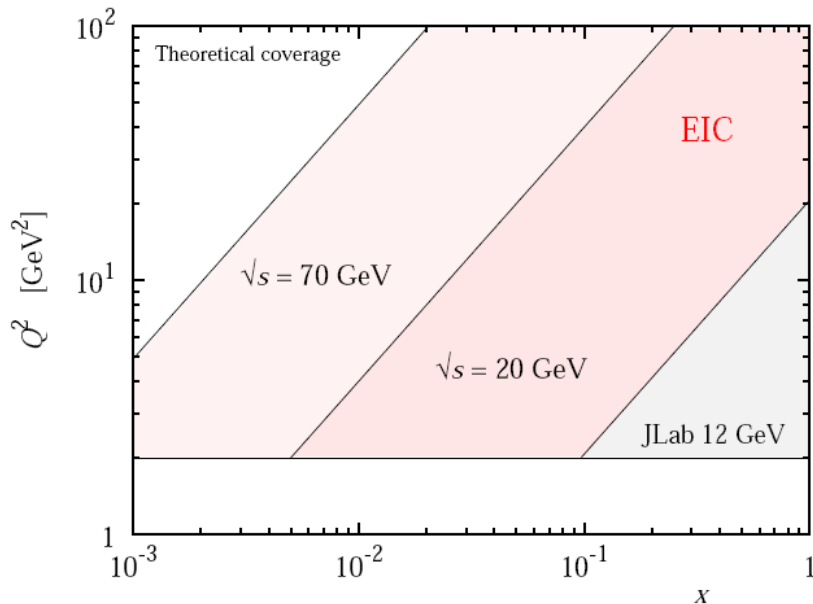
- **Lecture IV**

Evolution of TMDs

# Exploring the nucleon: a fundamental quest



# Nucleon landscape



**Nucleon is a many body dynamical system of quarks and gluons**

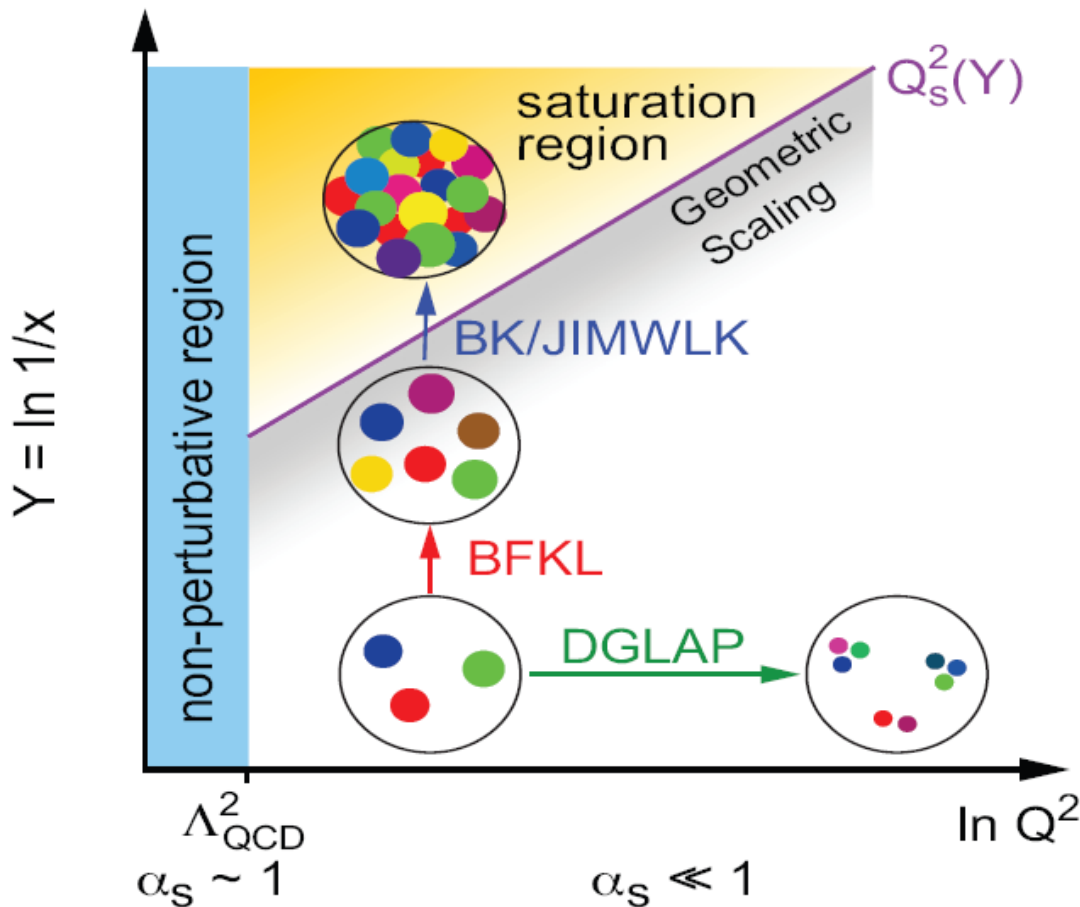
Changing  $x$  we probe different aspects of nucleon wave function

How **partons move** and how they are distributed in **space** is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

These distributions are also referred to as **3D (three-dimensional) distributions**

# Nucleon landscape



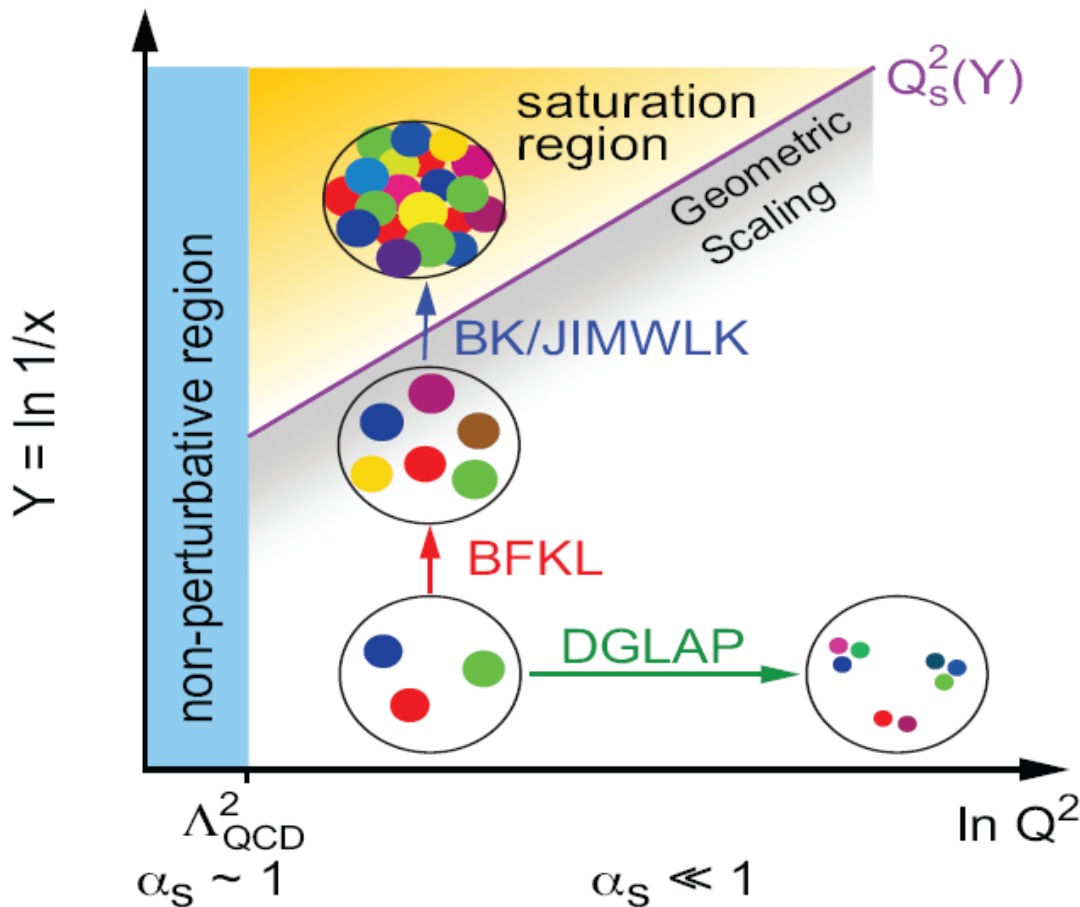
Virtual photon serves as a microscopic probe of the nucleon:

Larger  $Q^2$  probe smaller distances - DGLAP evolution

$$\lambda \sim 1/Q$$

Plot from EIC whitepaper

# Nucleon landscape

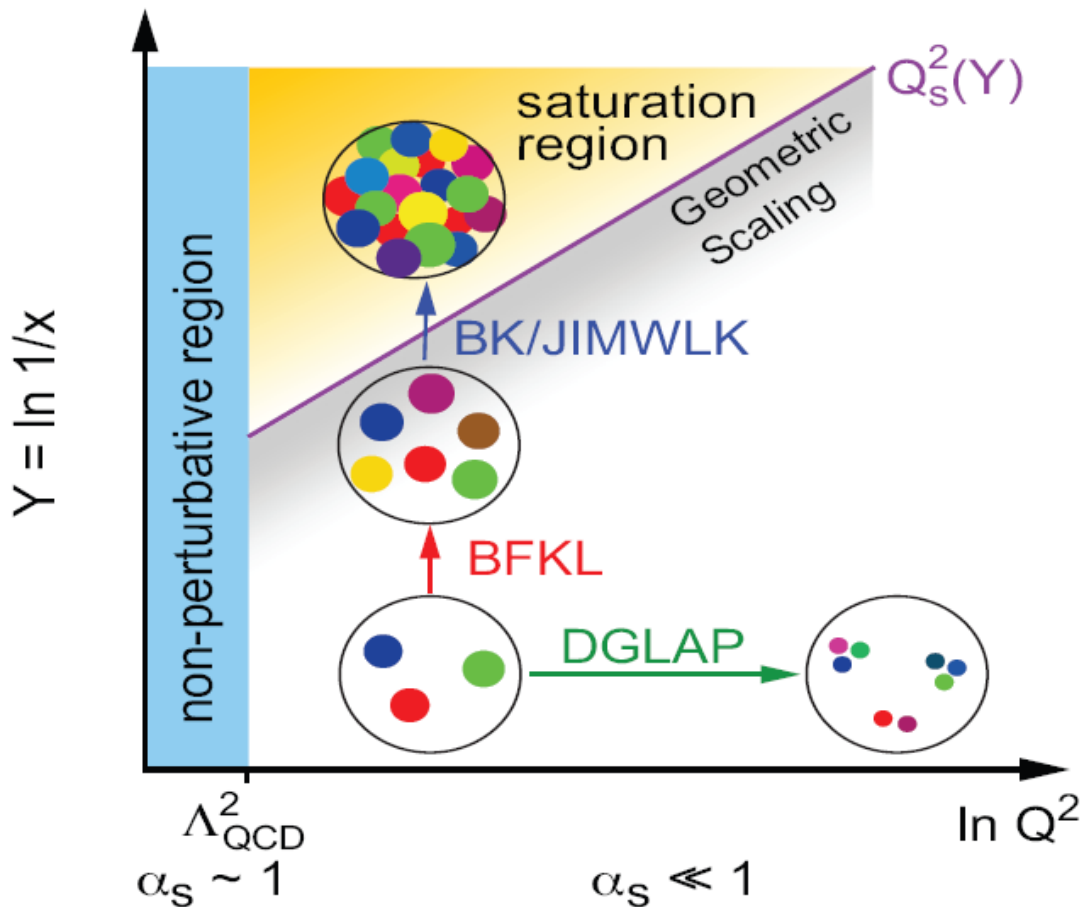


Virtual photon serves as a microscopic probe of the nucleon:

Fixing  $Q^2$  and changing the energy we probe BFKL evolution

Plot from EIC whitepaper

# Nucleon landscape



Virtual photon serves as a microscopic probe of the nucleon:

Recombination of gluons leads to non linear effects – BK/JIMWLK evolution and phenomenon of saturation. Dilute vs dense regime of QCD.

Plot from EIC whitepaper

“Experiments with spin have killed more theories than any other single physical parameter”

Elliot Leader, Spin in Particle Physics, Cambridge U. Press (2001)

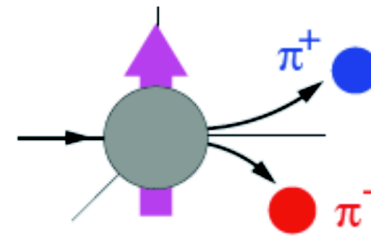
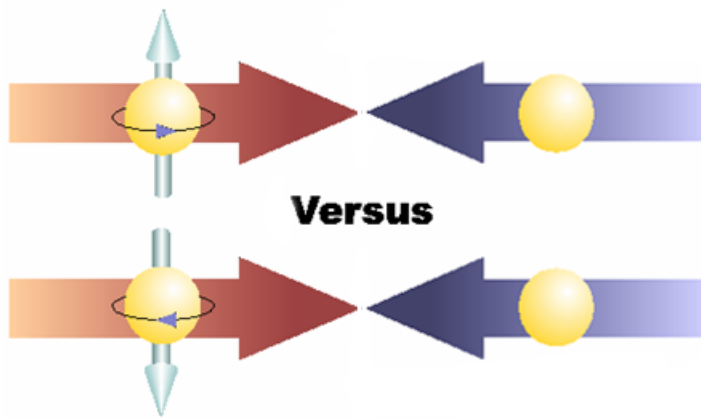
“Polarisation data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.”

J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hadronic Processes, St. Croix, Virgin Islands (1987).

# History

# Spin and QCD

Consider  $A_N$  in hadron hadron collision:

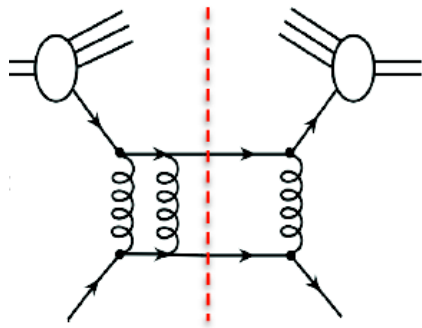


$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

# Spin and QCD

QCD had a very simple prediction:

Helicity flip is proportional to the small mass of the quark, thus

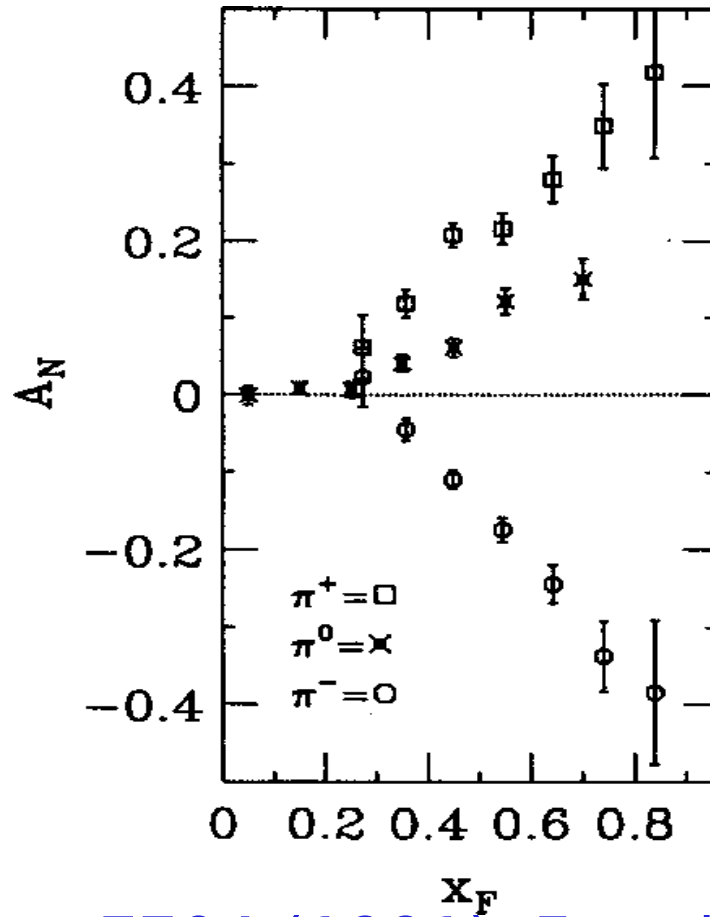


$$\propto \alpha_s \frac{m_q}{p_T}$$

$$A_N \simeq 0.001$$

Kane, Pumplin and Repko (1978)

Experiment proved this prediction wrong

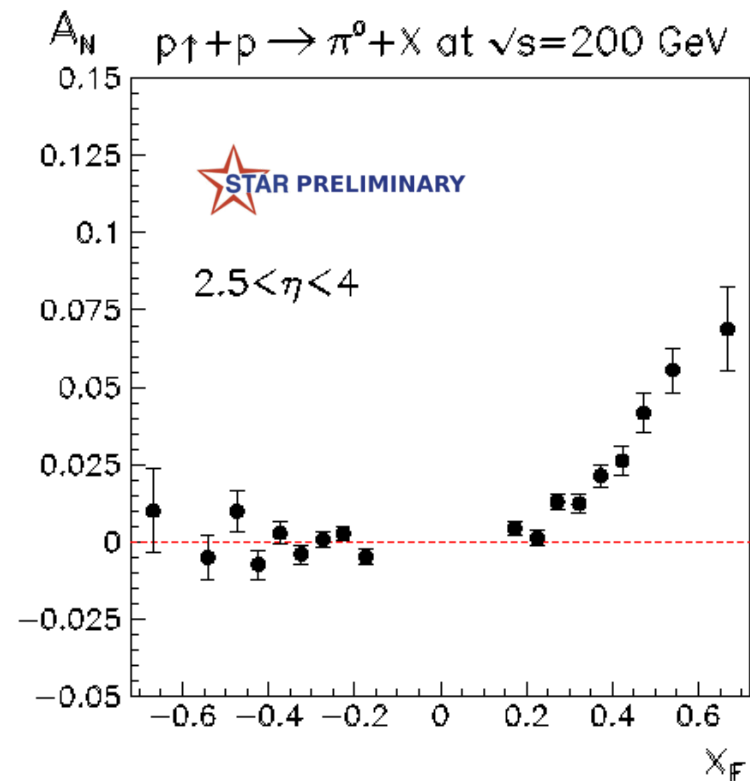
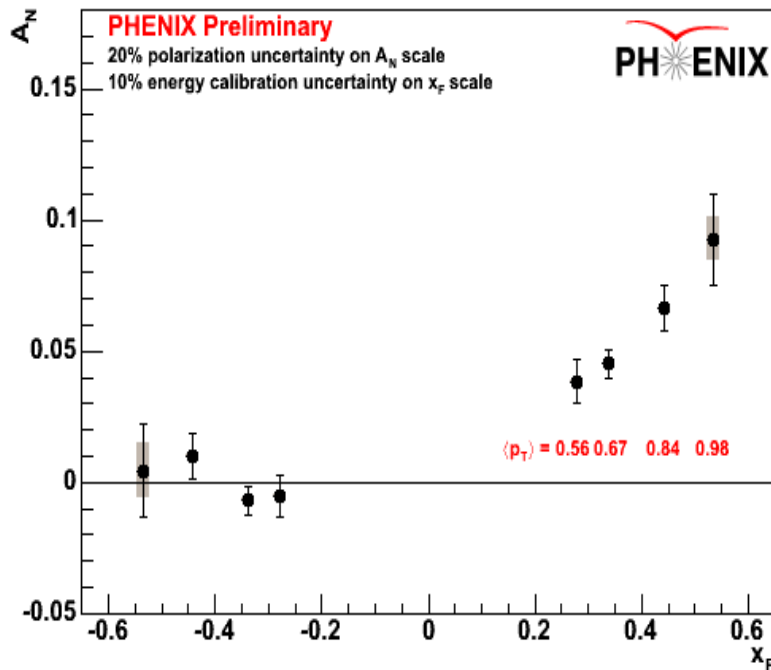


E704 (1991), Fermilab

$$A_N \simeq 40\%$$
$$\sqrt{s} = 19.1 \text{ (GeV)}$$

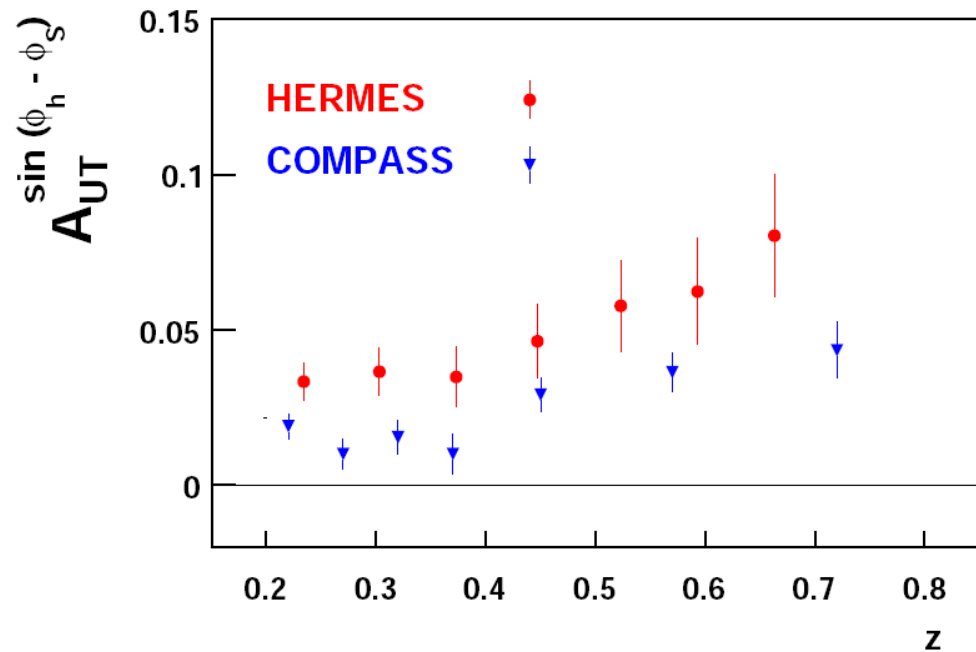
## Asymmetry survives with energy

$$\sqrt{s} = 62 \text{ GeV}$$



**RHIC: STAR, BRAHMS and PHENIX**

Asymmetry survives with energy



HERMES and COMPASS

# Failure of QCD?



# Not at all: better understanding of QCD



# Not at all: better understanding of QCD

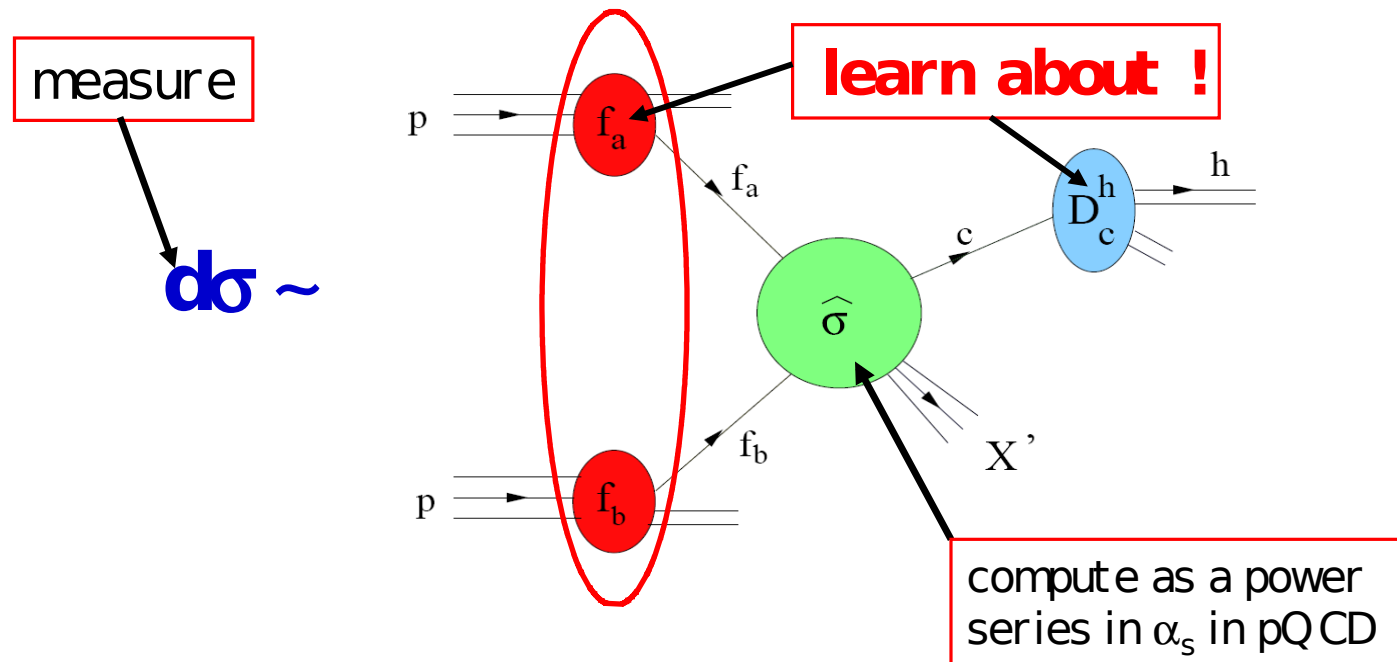


Eventually ....., but what happened in 1995-2000s?

# History of understanding

# QCD factorization

- Factorization of short-distance and long-distance physics

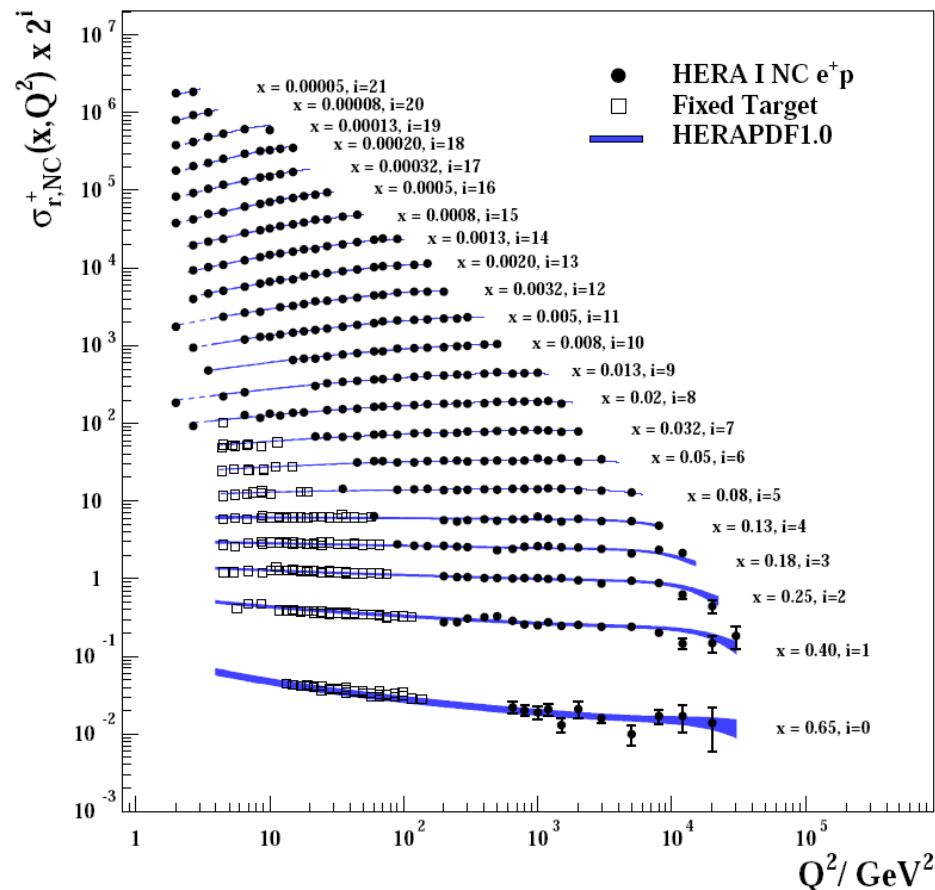


$$\sigma(P_h, S) \propto \underbrace{f_a(x_a, \mu^2)}_{\text{Universal}} \otimes \underbrace{f_b(x_b, \mu^2)}_{\text{Universal}} \otimes \underbrace{\hat{\sigma}_{ab \rightarrow c}}_{\text{calculable}} \otimes \underbrace{D_{h/c}(z_c, \mu^2)}_{\text{Universal}}$$

# Success of QCD factorization

- Universality of PDFs: mapped in one process (say DIS), used in other process

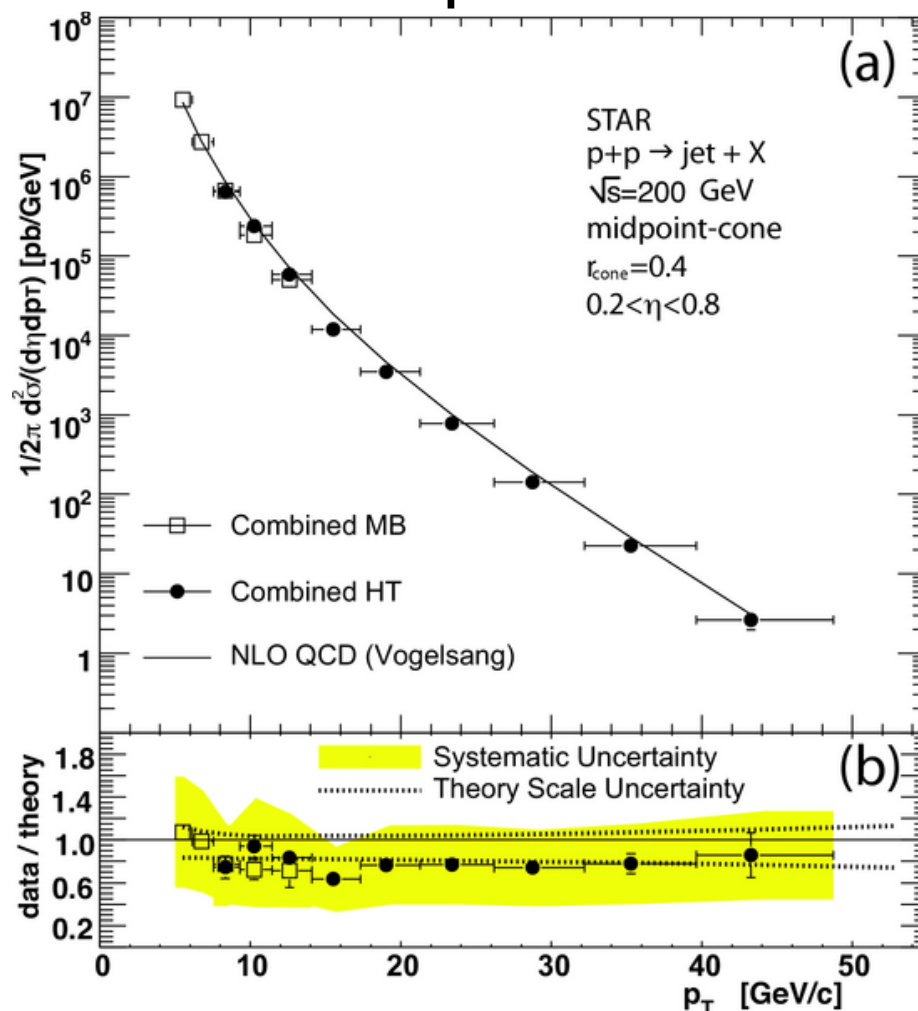
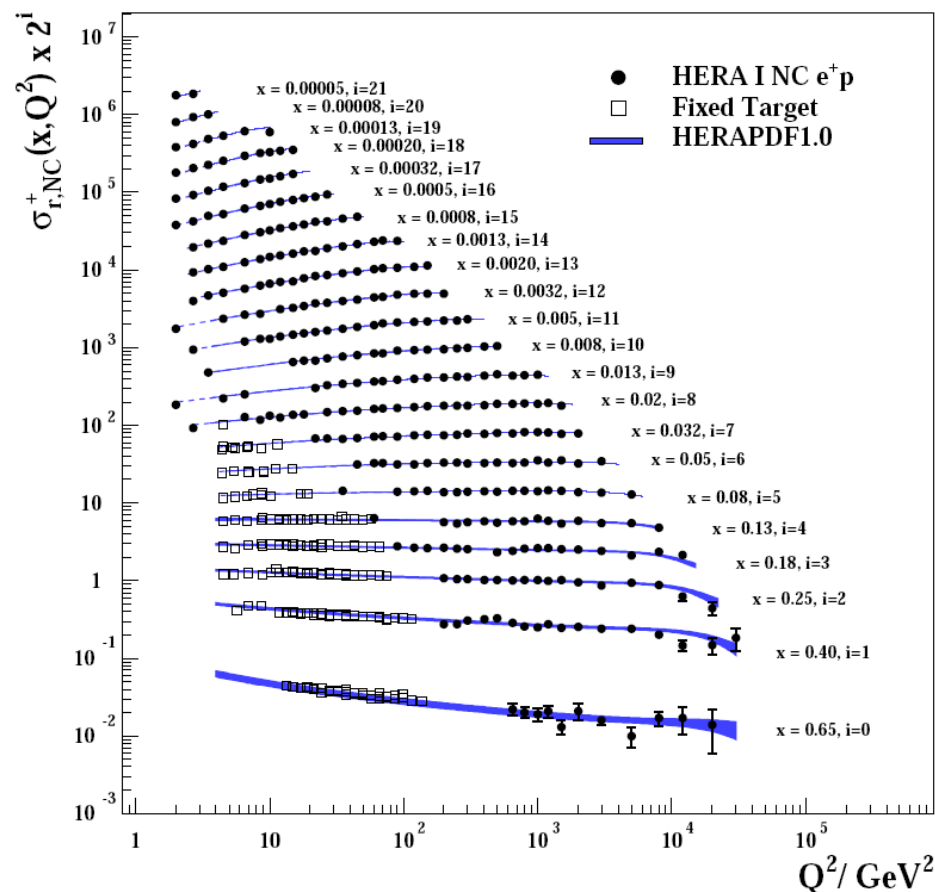
H1 and ZEUS



# Success of QCD factorization

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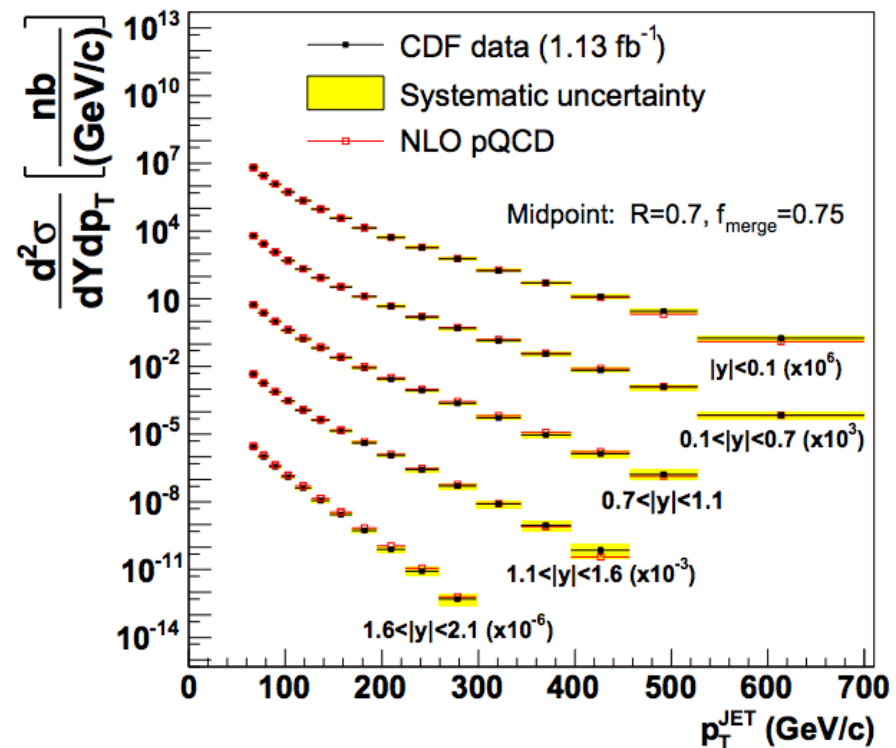
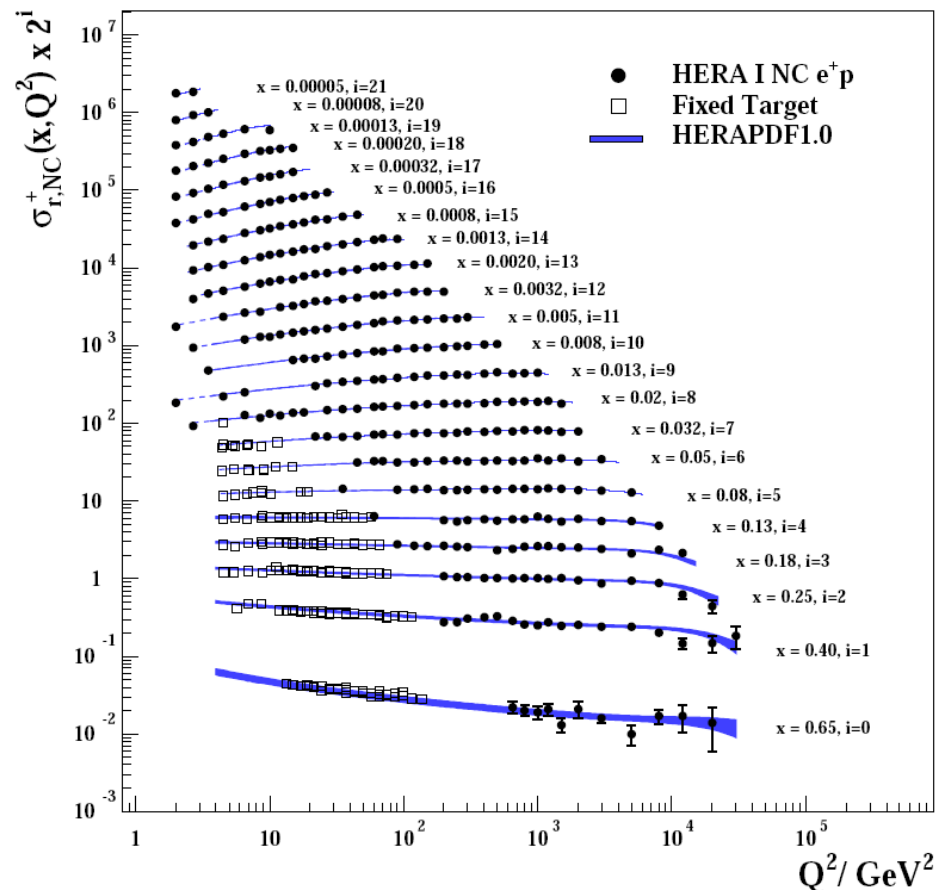
H1 and ZEUS



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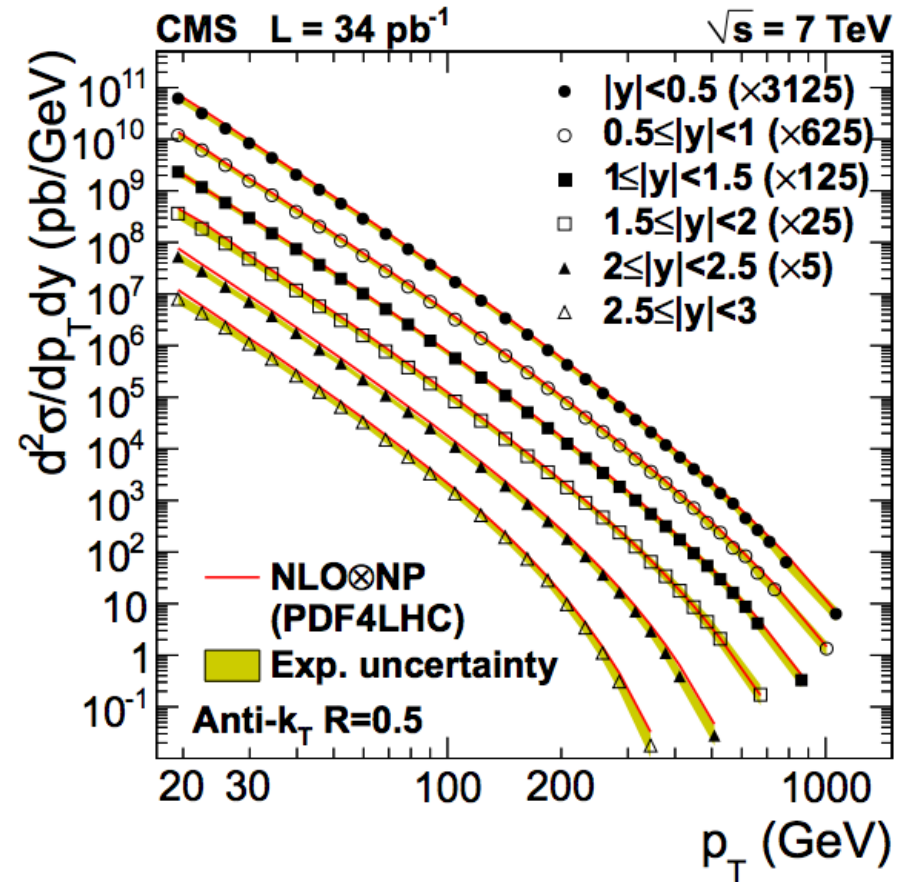
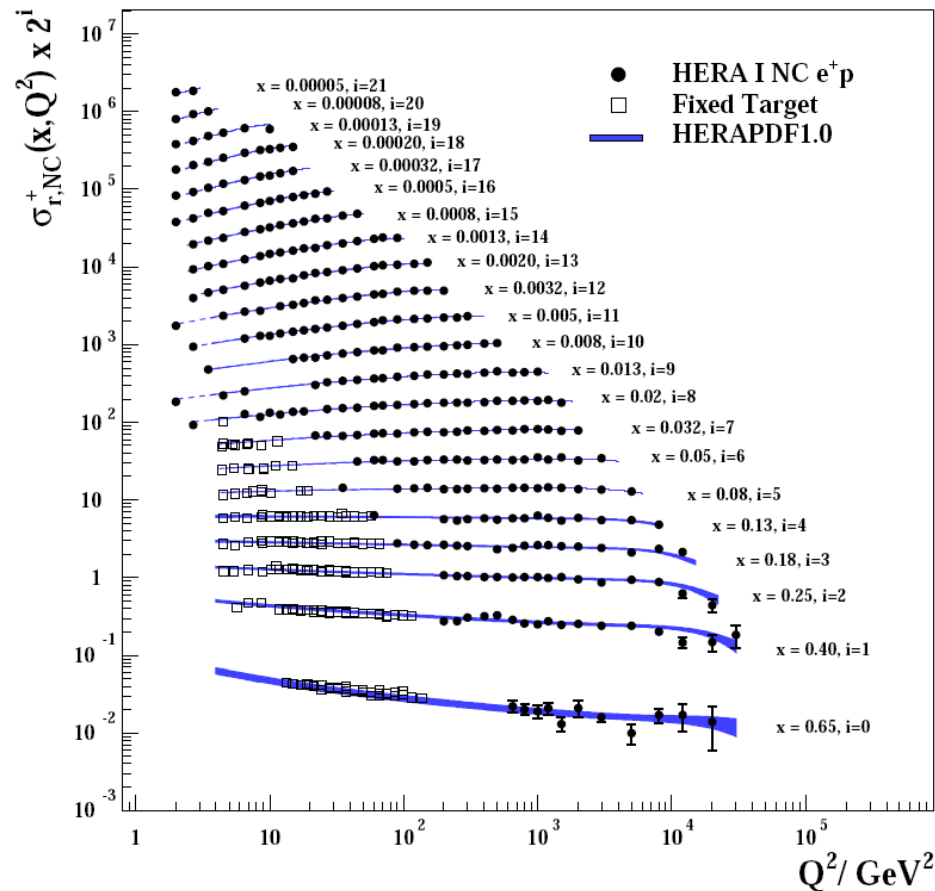
H1 and ZEUS



# Success of QCD factorization

- Universality of PDFs: mapped in one process (say DIS), used in other process

H1 and ZEUS



# The birth of TMDs (as phenomenological quantities):

D. Sivers, PRD 41 (1990) 83

$$G_{a/p}(x; \mu^2) \rightarrow G_{a/p}(x, \mathbf{k}_T; \mu^2)$$

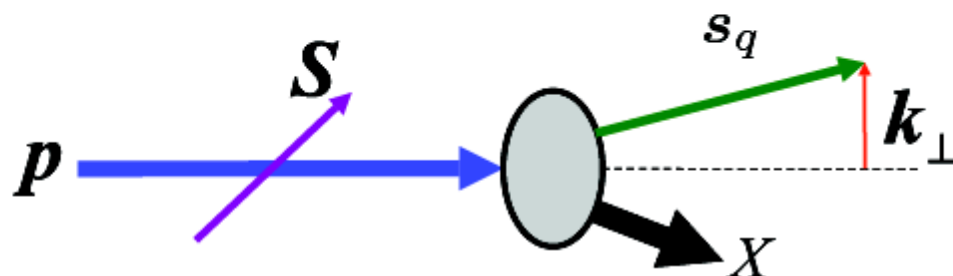
The relevance of the transverse momentum for the asymmetry can be seen from the venerable Chou-Yang<sup>1</sup> model of the constituent structure of a transversely polarized proton. If we assume a **correlation between the spin of the proton and the orbital motion of its constituents**, Chou and Yang showed the existence of a nontrivial  $A_N$  in elastic scattering. The coherent dynamics which correlates the spin of the proton with the orbital angular momentum of the quarks and gluons can also produce a constituent-level asymmetry in transverse momentum:

$$\begin{aligned} \Delta^N G_{a/p(\uparrow)}(x, \mathbf{k}_T; \mu^2) &= \sum_h [G_{a(h)/p(\uparrow)}(x, \mathbf{k}_T; \mu^2) - G_{a(h)/p(\downarrow)}(x, \mathbf{k}_T; \mu^2)] \\ &= \sum_h [G_{a(h)/p(\uparrow)}(x, \mathbf{k}_T; \mu^2) - G_{a(h)/p(\uparrow)}(x, -\mathbf{k}_T; \mu^2)] \end{aligned}$$

<sup>1</sup> T. T. Chou and C. N. Yang, Nucl. Phys. B107, 1 (1976)

$$A_N \left[ E \frac{d^3\sigma}{d^3p} (pp \rightarrow mX) \right] \simeq \sum_{ab \rightarrow cd} \int d^2\mathbf{k}_T^a dx_a \int d^2\mathbf{k}_T^b dx_b \int d^2\mathbf{k}_{TC} \frac{dx_c}{x_c^2} \Delta^N G_{a/p\uparrow}(x_a, \mathbf{k}_T^a; \mu^2) \\ \times G_{b/p}(x_b, \mathbf{k}_T^b; \mu^2) D_{m/c}(x_c, \mathbf{k}_T^c; \mu^2) \times \tilde{s} \frac{d\sigma}{d\tilde{t}}(ab \rightarrow cd) \delta(\tilde{s} + \tilde{t} + \tilde{u})$$

.... this equation corresponds to a probabilistic formula in the original spirit of the parton model where the unknown soft nonperturbative dynamics have been absorbed into the specification of the density  $\Delta^N G$  ...



$$f_{q/p, \mathbf{S}}(x, \mathbf{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ = f_{q/p}(x, k_\perp) - \frac{k_\perp}{M} f_{1T}^\perp(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

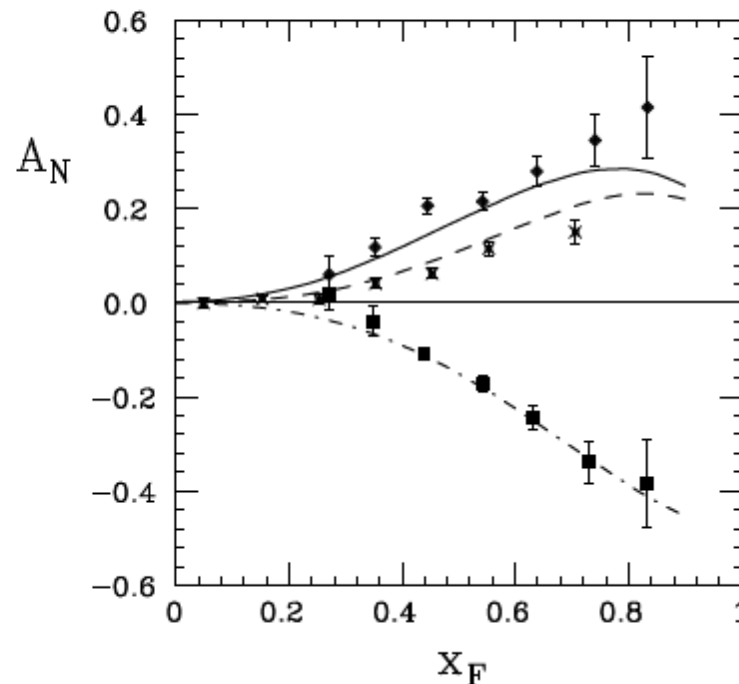
Anselmino, Transversity 2014

# early $A_N$ phenomenology with Sivers function

(M.A., M. Boglione and F. Murgia, PL B 362 (1995) 164)

$$\frac{E_\pi d\sigma^{p^\uparrow p \rightarrow \pi X}}{d^3 p_\pi} \sim \frac{1}{2} \sum_{a,b,c,d} \sum_{\lambda_a, \lambda'_a; \lambda_b, \lambda_c, \lambda'_c; \lambda_d} \int d^2 \mathbf{k}_{\perp a} dx_a dx_b \frac{1}{z}$$

$$\rho_{\lambda_a, \lambda'_a}^{a/p^\uparrow} \hat{f}_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) f_{b/p}(x_b) \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} \hat{M}_{\lambda'_c, \lambda_d; \lambda'_a, \lambda_b}^* D_{\pi/c}^{\lambda_c, \lambda'_c}(z)$$



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Asymmetry comes from modulation of the **initial** distribution function (D.Sivers 1990)

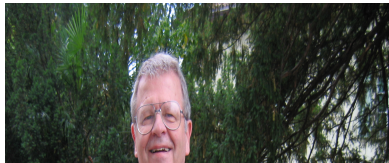


Asymmetry comes from modulation in **final** state fragmentation (J.Collins 1993)



Asymmetry comes from modulation of the **initial** distribution function (D.Sivers 1990)

Asymmetry comes from modulation in **final** state fragmentation (J.Collins 1993)



**Sivers effect forbidden by  
time reversal invariance  
(Collins 1993)**

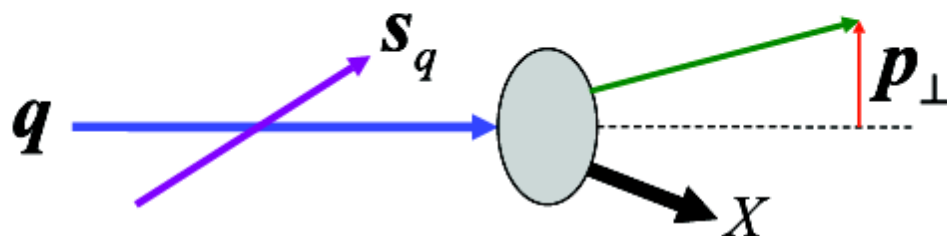


Sivers suggested that the  $k_{\perp}$  distribution of the quark could have an azimuthal asymmetry when the initial hadron has transverse polarization. However, such an asymmetry is prohibited because QCD is time-reversal invariant....

# Collins fragmentation function

Nucl. Phys. B396 (1993) 161

It is shown that the azimuthal dependence of the distribution of hadrons in a quark jet is a probe of the transverse spin of the quark initiating the jet. This results in a new spin-dependent fragmentation function that acts at the twist-2 level.



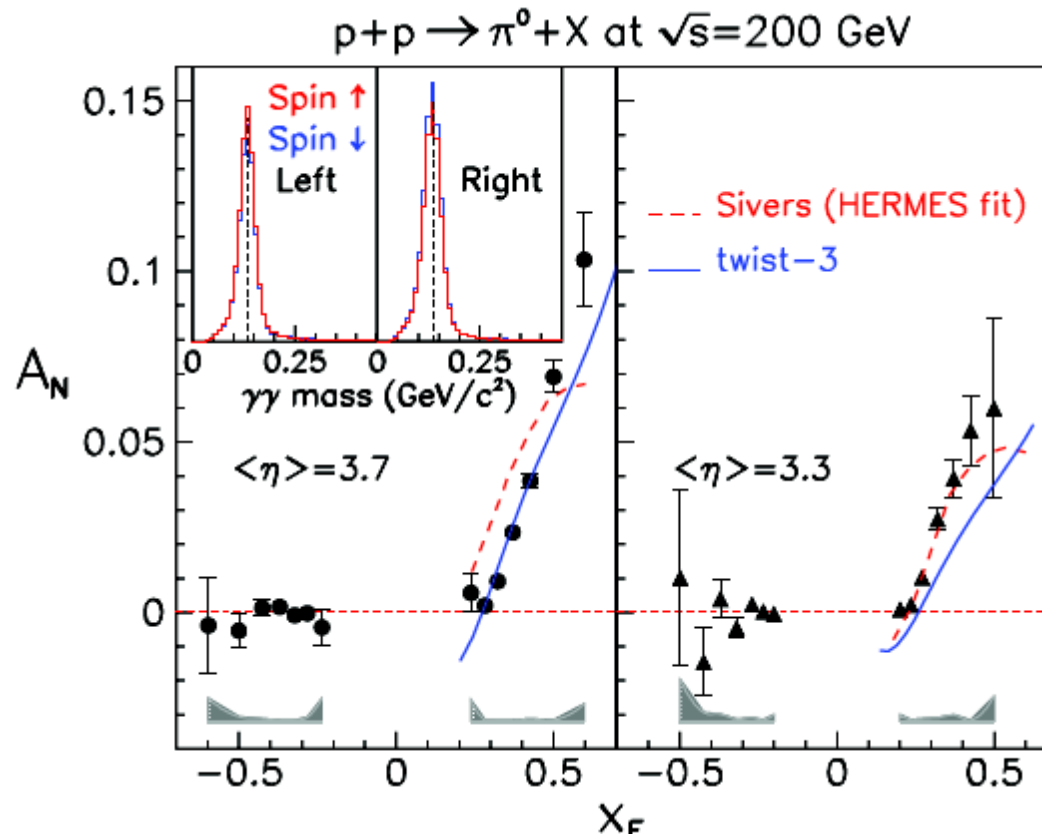
Collins  
function

$$\begin{aligned}
 D_{h/q,s_q}(z, \mathbf{p}_\perp) &= D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \\
 &= D_{h/q}(z, p_\perp) + \frac{p_\perp}{zM_h} H_1^{\perp q}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp)
 \end{aligned}$$

Anselmino, Transversity 2014

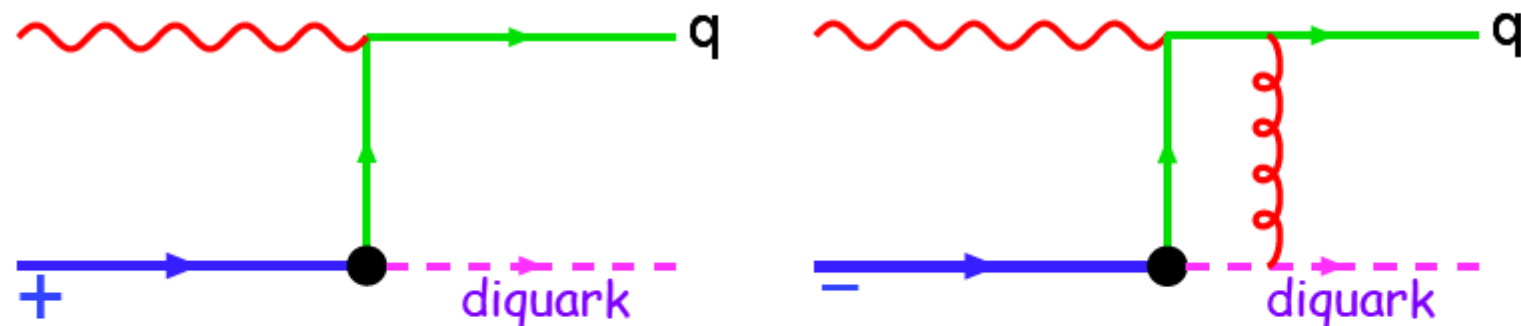
# Phenomenology never stopped...

Prediction of AN with Sivers effect



gauge links have physical consequences;  
quark models for non vanishing Sivers function,

### SIDIS final state interactions



Brodsky, Hwang, Schmidt, PL B530 (2002) 99 - Collins, PL B536 (2002) 43

An earlier proof that the Sivers asymmetry vanishes because of time-reversal invariance is invalidated by the path-ordered exponential of the gluon field in the operator definition of parton densities. Instead, the time-reversal argument shows that the Sivers asymmetry is reversed in sign in hadron-induced hard processes (e.g., Drell-Yan), thereby violating naive universality of parton densities. Previous phenomenology with time-reversal-odd parton densities is therefore validated.

$$[f_{1T}^{q\perp}]_{\text{SIDIS}} = -[f_{1T}^{q\perp}]_{\text{DY}}$$

# Another way to explain the asymmetry

$$\sigma(Q, \vec{s}) \propto \left| \begin{array}{c} \text{Diagram 1} \\ + \text{Diagram 2} \\ + \text{Diagram 3} \\ + \dots \end{array} \right|^2$$

The diagrams show multi-parton correlations. The first diagram has an incoming line labeled  $p, \vec{s}$  and an outgoing line labeled  $k$ . A vertex is labeled  $t \sim 1/Q$ . The subsequent diagrams show more complex internal structures with additional lines and vertices, representing higher-order correlations.

Multi parton correlations contribute to the cross section.

These correlations are called  
[Efremov-Teryaev-Qiu-Stermann](#) matrix elements,  
 They appear at twist-3 level in cross section.

$$\sigma = \sigma^{LT} + \frac{Q_s}{Q} \sigma^{HT} + \dots$$

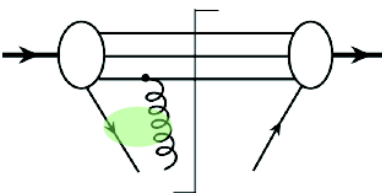
$$= H^{LT} \otimes f_2 \otimes f_2 + \frac{Q_s}{Q} H^{HT} \otimes f_3 \otimes f_2 + \dots$$

If only one large scale is present in the process, then

$$\begin{aligned}
 A_N &\propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp) \\
 &\propto T^{(3)}(x, x, S_\perp) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x, S_\perp) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \dots
 \end{aligned}$$

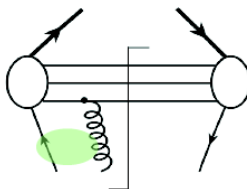
Leading power cancels

Twist-3 parton correlation functions

$$T^{(3)}(x, x, S_\perp) \propto$$


Qiu-Sterman 1991

Twist-3 parton fragmentation functions

$$D^{(3)}(z, z) \propto$$


Kang, Yuan, Zhou 2010

No probability interpretation!

## TMD formalism:

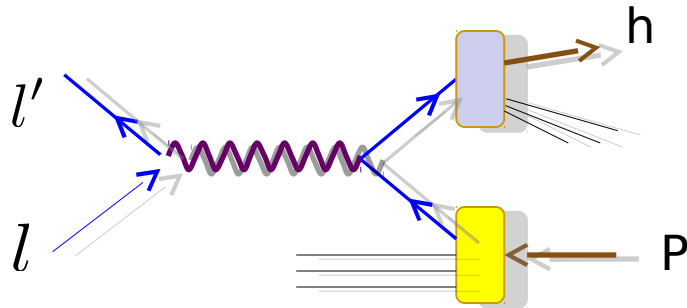
Sivers, Collins effects, other functions

## Twist-3 functions:

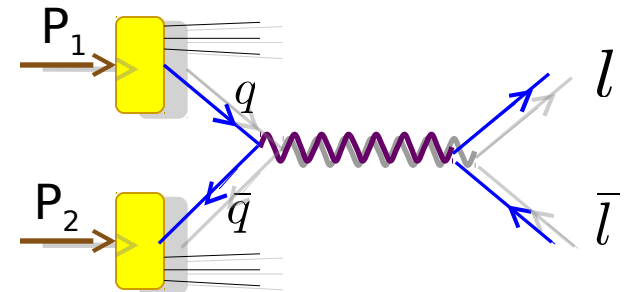
Qiu-Sterman matrix elements

Are these two formalisms “competing” with each other?  
Are there relations between these functions?

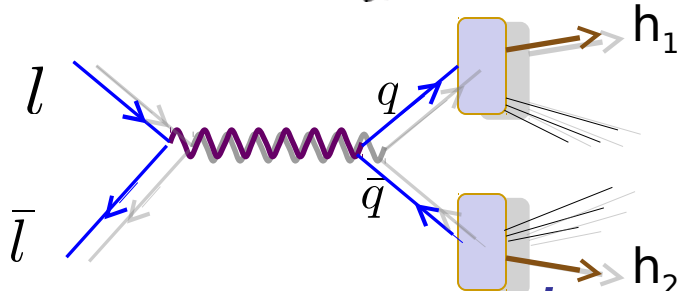
# Other experiments, other processes:



*SIDIS*



*Drell-Yan*



*$e^-e^+$  to pions*

hard scattering



Partonic Fragmentation

Partonic Distributions

# Modern view on hadron structure

# I: relation of TMD and Twist-3

Two observed scales

$Q_1 \sim \Lambda_{QCD}$  sensitive to  
parton's  
transverse motion

$Q_2, \dots \gg \Lambda_{QCD}$  ensures pQCD

$$f(x, k_{\perp}; Q^2)$$

TMD distributions

One observed momentum scale

$$Q_1, Q_2, \dots \gg \Lambda_{QCD}$$

$$f(x; Q^2)$$

Collinear distributions

TMD distributions

$$f(x, k_{\perp}; Q^2)$$

Evolution CSS

Collins Soper Sterman 1985  
Ji Ma Yuan 2004  
Collins 2011

Collinear distributions

$$f(x; Q^2)$$

Evolution DGLAP

TMD distributions

$$f(x, k_{\perp}; Q^2)$$

Evolution CSS

Collins Soper Serman 1985

Ji Ma Yuan 2004

Collins 2011

$f(x; Q^2)$  is an ingredient with  
corresponding DGLAP

Collinear distributions

$$f(x; Q^2)$$

Evolution DGLAP

TMD distributions

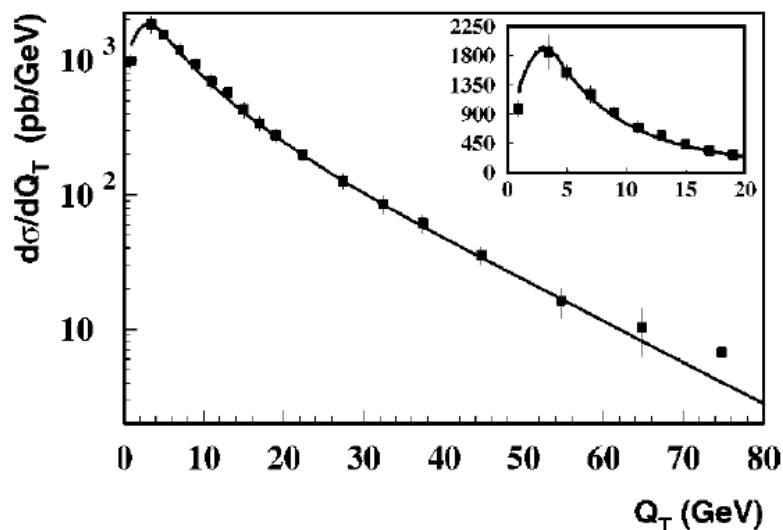
$$f(x, k_{\perp}; Q^2)$$

Phenomenology:

A lot of different functions

Mainly LO (tree level) for spin dependent

Very advanced in unpolarised case



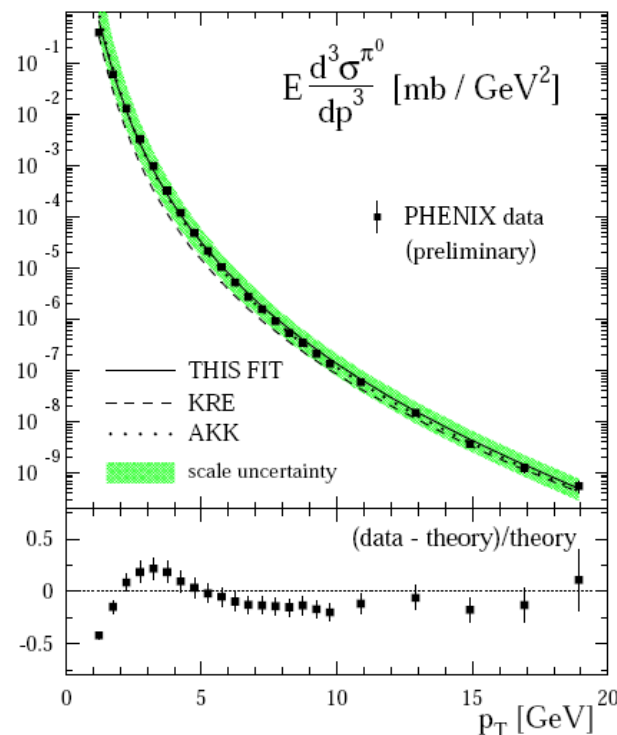
Brock, Landry, Nadolsky, Yuan 2003  
Qiu, Zhang 2001

Collinear distributions

$$f(x; Q^2)$$

Phenomenology:

NLO, NNLO ...



De Florian, Sassot, Stratmann 2007  
(DSS)

TMD distributions

$$f(x, k_{\perp}; Q^2)$$

Phenomenology:

A lot of different functions

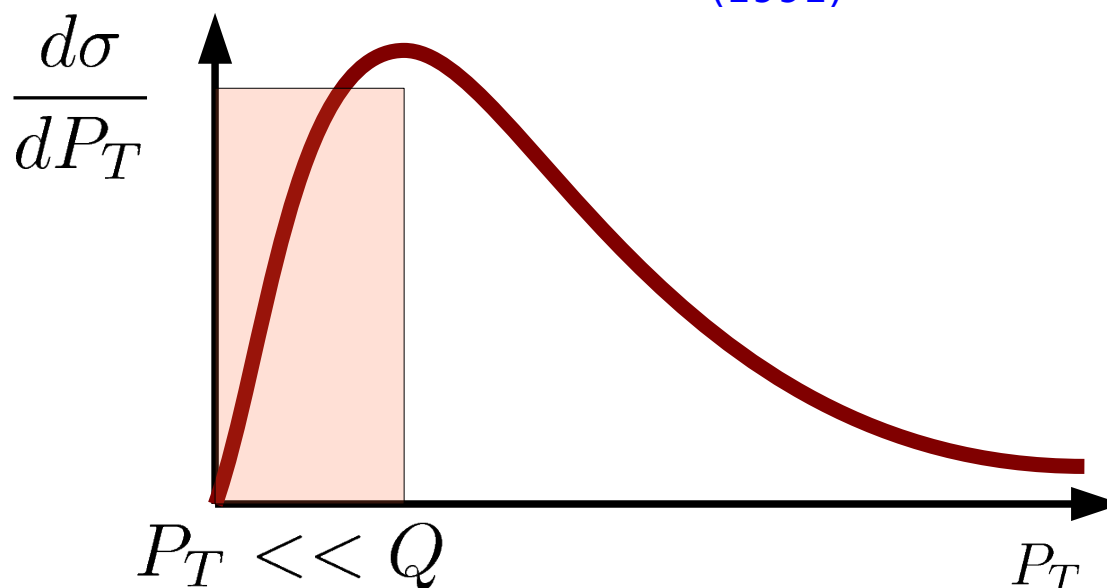
Collinear distributions

$$f(x; Q^2)$$

Beyond leading twist:

Twist-3 matrix elements

Efremov Teryaev (1982), Qiu, Sterman (1991)



Mulders, Tangerman 1995

Boer, Mulders 1998

TMD distributions

$$f(x, k_{\perp}; Q^2)$$

Phenomenology:

A lot of different functions

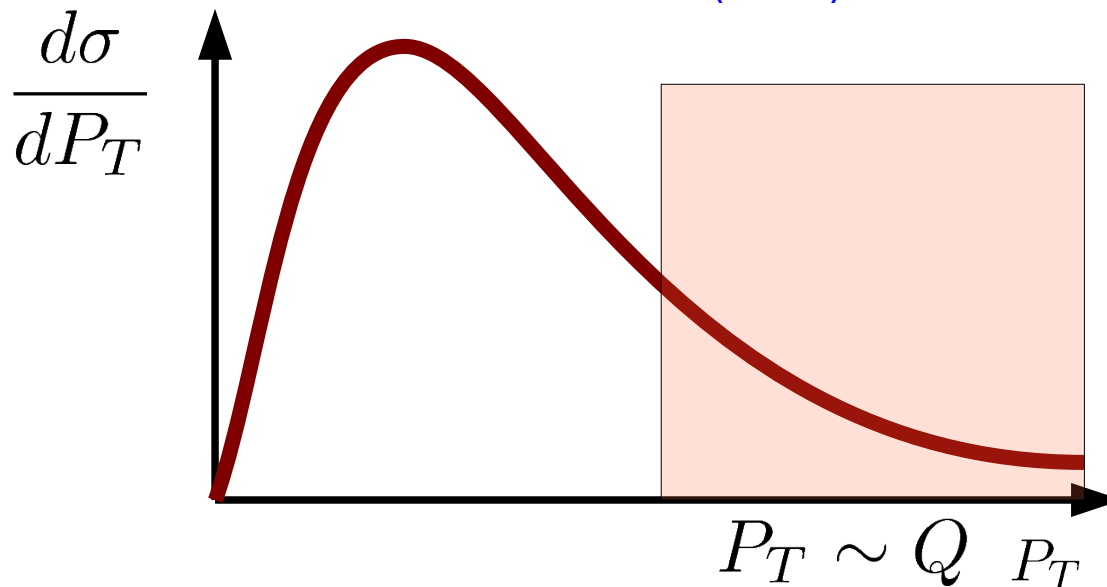
Collinear distributions

$$f(x; Q^2)$$

Beyond leading twist:

Twist-3 matrix elements

[Efremov Teryaev \(1982\)](#), [Qiu, Serman \(1991\)](#)



TMD distributions

$$f(x, k_{\perp}; Q^2)$$

Phenomenology:

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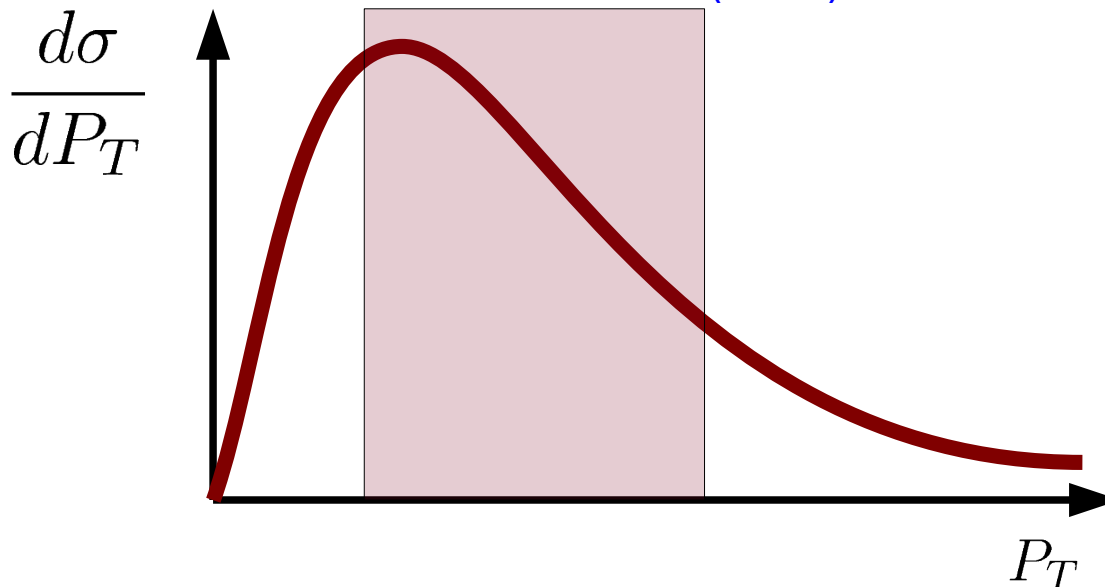
Collinear distributions

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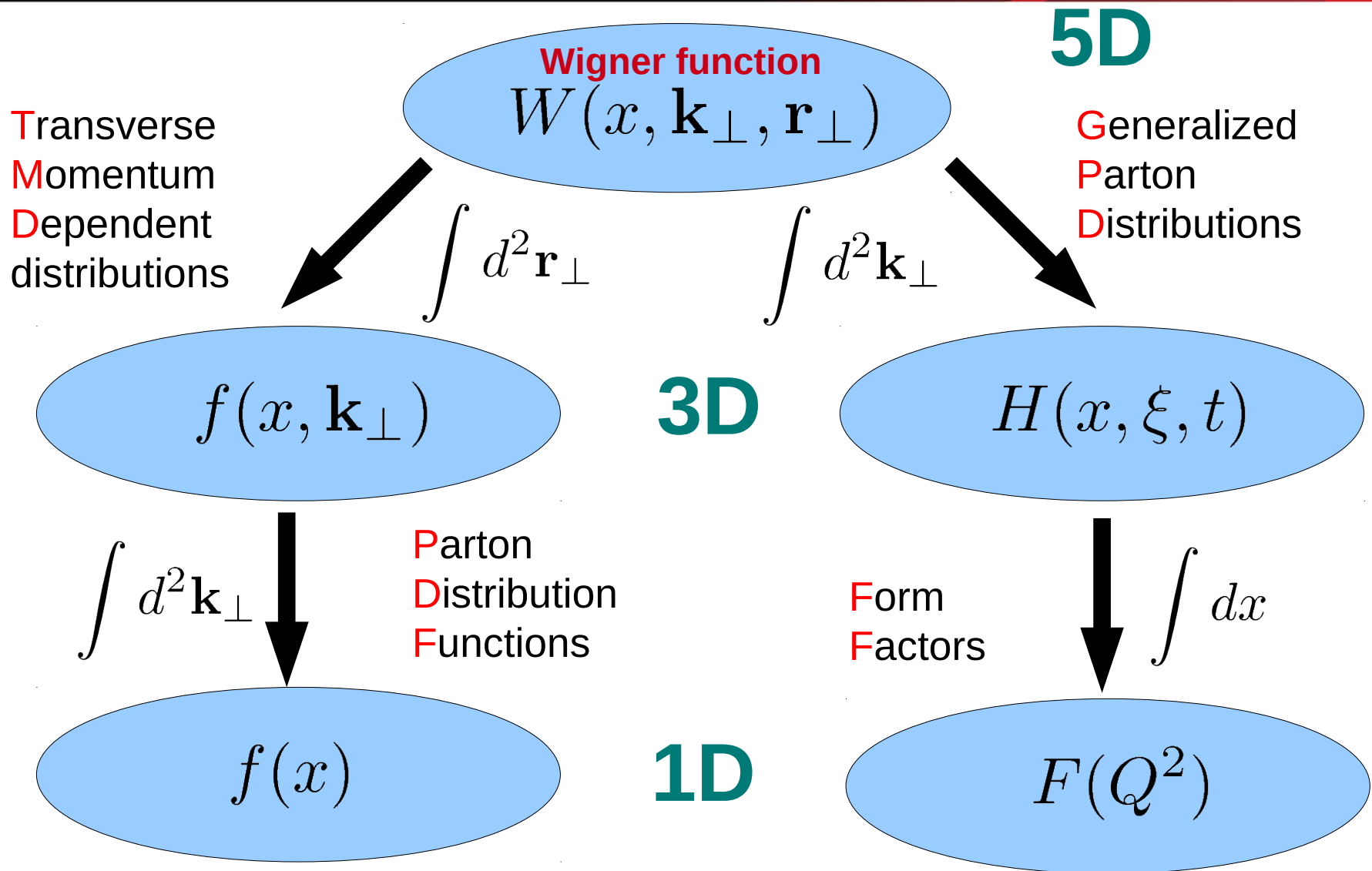


Intermediate region, both formalisms are applicable and related

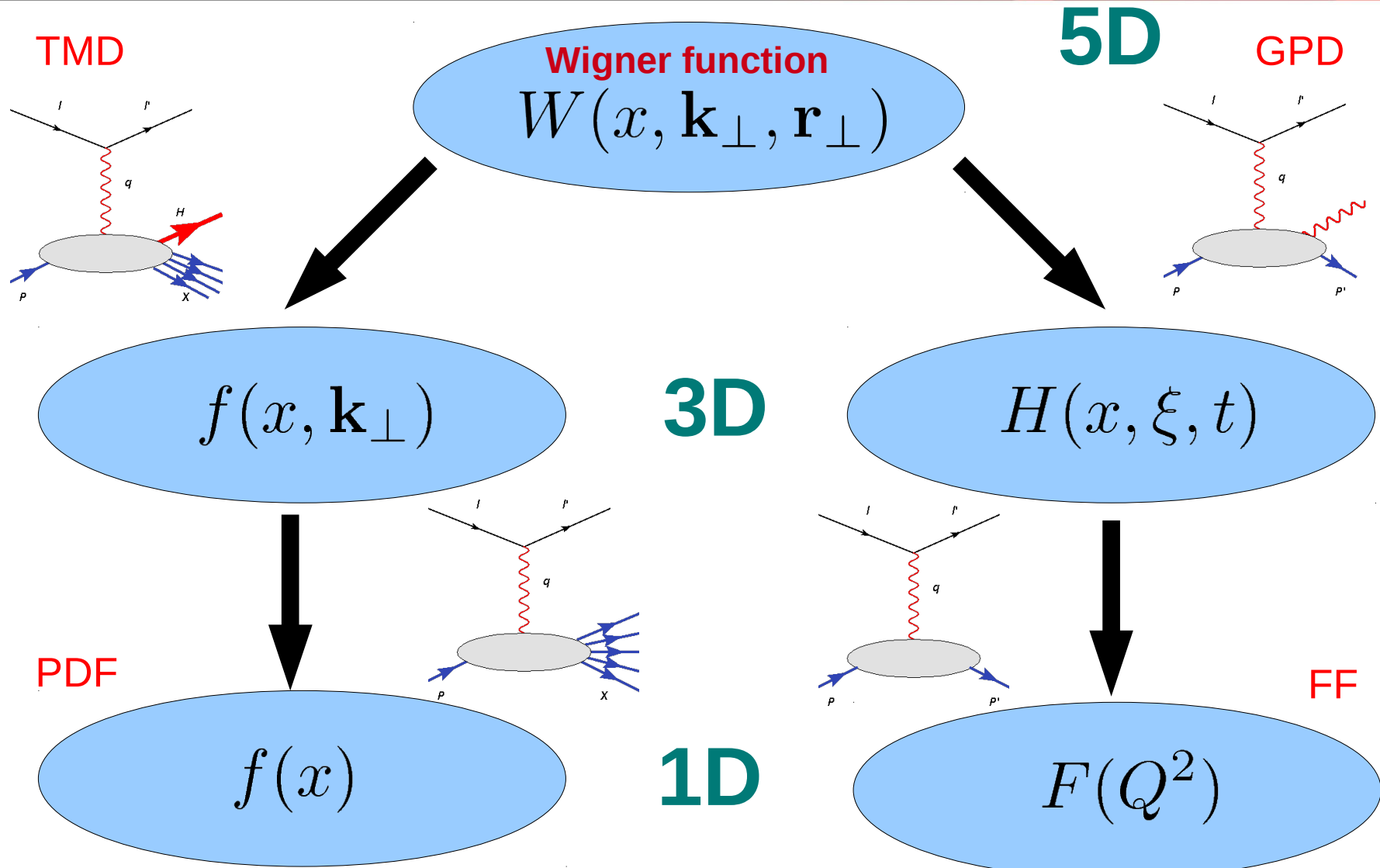
Ji, Qiu, Vogelsang, Yuan (2006) etc

## II: Generalization of TMD formalism

# Unified View of Nucleon Structure

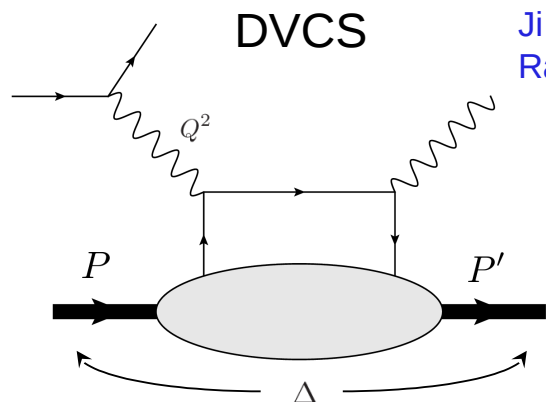


# Unified View of Nucleon Structure



Particular processes to study. Polarization is required!

# GPDs



Ji (1997)  
Radyushkin (1997)

$Q^2$  ensures hard scale, pointlike interaction

$\Delta = P' - P$  momentum transfer can be varied independently

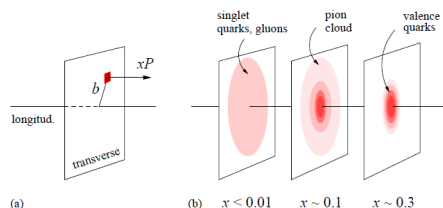
Connection to 3D structure

Burkardt (2000)  
Burkardt (2003)

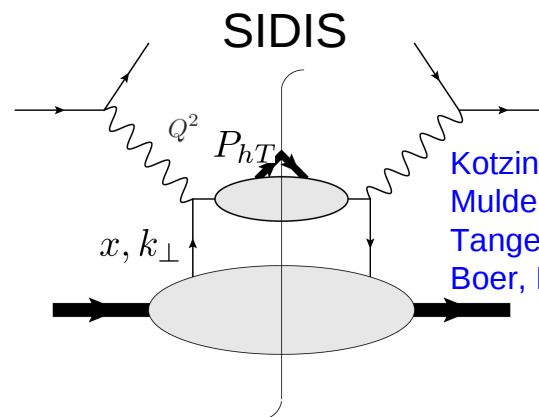
$$\rho(x, \vec{r}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

Drell-Yan frame  $\Delta^+ = 0$

Weiss (2009)



# TMDs



Kotzinian (1995),  
Mulders,  
Tangerman (1995),  
Boer, Mulders (1998)

$Q^2$  ensures hard scale, pointlike interaction

$P_{hT}$  final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)  
Collins (2011)

$$\tilde{f}(x, \vec{b}_T) = \int d^2 k_\perp e^{-i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$$

AP (2012)

